

Density correlations in ultracold atomic Fermi gases

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We investigate density fluctuations in a coherent ensemble of interacting fermionic atoms. Adapting the concept of full counting statistics, well-known from quantum optics and mesoscopic electron transport, we study second-order as well as higher-order correlators of density fluctuations. Using the mean-field BCS state to describe the whole interval between the BCS limit and the BEC limit, we obtain an exact expression for the cumulant-generating function of the density fluctuations of an atomic cloud. In the two-dimensional case, we obtain a closed analytical expression. Poissonian fluctuations of a molecular condensate on the BEC side are strongly suppressed on the BCS side. The size of the fluctuations in the BCS limit is a direct measure of the pairing potential. We also discuss the BEC-BCS crossover of the third cumulant and the temperature dependence of the second cumulant.

I. INTRODUCTION

Following the successful creation of Bose-Einstein condensates (BECs) in ultracold atomic clouds [1], recently ultracold Fermionic clouds have been produced [2, 3, 4, 5, 6]. This has attracted a lot of attention both theoretically and experimentally, especially due to the ability to tune the mutual interaction between atoms via a Fano-Feshbach resonance. The unique opportunity to study the crossover from weak attractive to strong attractive interactions in one and the same system makes this interesting from a fundamental many-body point of view.

Theoretically, fermionic systems with weak attractive interaction are superfluids and as such described by the Bardeen-Cooper-Schrieffer (BCS) theory [7]. This theory can also describe the limit of stronger attractive interaction [8, 9, 10], in which a BEC of molecules is formed. In the crossover regime, the long-range nature of the interaction makes the BCS theory less accurate [11].

Recently, several experiments studied the strongly interacting BEC-BCS crossover regime using spin mixtures of ultracold fermionic gases (see [12] for a recent review). Measurements of the interaction energy of an ultracold fermionic gas near a Feshbach resonance were made, studying the impact of the interaction on the time of flight expansion [13]. Experimental investigation of collective excitations showed a strong dependence on the coupling strength [14, 15]. Moreover, the condensation [16, 17] and the spatial correlations [18] of the fermionic atom pairs were observed in the full crossover regime. The pairing gap was measured directly via a spectroscopic technique in the whole crossover region [19]. Remarkably, the gap values are in relatively good agreement with the BCS model in the whole region.

The use of noise correlations to probe the many-body states of ultracold atoms was proposed in [20] (see also [21]). Correlation measurements can be applied to detect phase coherence in mesoscopic superpositions [22].

The density and spin structure factor for the BEC-BCS transition was calculated [23]. Interferometric measurement schemes of the spatial pairing order have been proposed based on the atom counting statistics in the output channels [24]. Pairing fluctuations of trapped Fermi gases have been studied in [25]. The number statistics of Fermi and Bose gases has also been investigated in [26, 27]. In recent experiments [28] the spatial structure of an atomic cloud has been directly observed (without the expansion used in most other experiments). This makes it possible to determine the density fluctuations either by repeating the experiment many times or by taking densities at different positions in a homogeneous system to extract the statistics. Atomic shot noise has been experimentally investigated both in bosonic and fermionic systems [29, 30, 31, 32, 33, 34, 35].

In this article we propose to use the full counting statistics of density fluctuations as a tool to gain access to the many-body nature of the ground state of a fermionic cloud in the BEC-BCS crossover regime (for other work on full counting statistics in ultracold atomic gases see [36] on the experimental and [37, 38, 39, 40] on the theoretical side). Our main result is a general expression for the particle number statistics of the mean-field BCS wave function. In the limiting cases, the statistics allows a straightforward interpretation. Deep in the molecular BEC limit the statistics is Poissonian, i.e. that of independent pairs of atoms. In the opposite limit, on the BCS side of the crossover, the fluctuations are strongly suppressed and reflect the particle-hole symmetry. The statistics in the crossover regime differs strongly from both the BCS and BEC limits and will be discussed based on several numerical results.

II. COUNTING STATISTICS OF DENSITY FLUCTUATIONS

We consider an atomic cloud with spatial distribution $n(x)$ and divide the system into bins, see Fig. 1. We are interested in the probability $P(N) = \langle \delta(N - \hat{N}) \rangle$ to find N atoms in a “bin” of the system, see Fig. 1. Here, $\hat{N} = \int_{V_{\text{bin}}} \hat{n}(x)$ is the bin number operator, V_{bin} is the bin volume, and $\hat{n}(x)$ denotes the density operator. The bin volume is assumed to be much smaller than the volume of the full system, but still much larger than the mean interparticle distance cubed. Hence, a single bin can be considered as a grand-canonical system with the surrounding atomic cloud serving as the reservoir.

In practice, it is more convenient to study the Fourier transform of $P(N)$, which is related to the cumulant generating function (CGF) $S(\chi)$ via

$$e^{-S(\chi)} = \sum_N e^{i\chi N} P(N) = \langle e^{i\chi \hat{N}} \rangle. \quad (1)$$

The cumulants C_n are defined in a standard way as $S(\chi) = -\sum_n C_n (i\chi)^n / n!$ and can be used to characterize the full counting statistics (FCS). We recall that $C_1 = \bar{N}$ is the average number of atoms in a bin, and $C_2 = \langle (N - \bar{N})^2 \rangle$ measures the width of the number distribution. The third cumulant is proportional to the skewness and, therefore, a measure of the asymmetry of the distribution function. As we will see later, the third cumulant in a fermionic system is related to deviations from particle-hole symmetry.

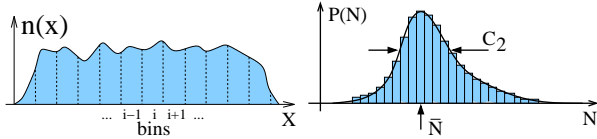


FIG. 1: (Color online) (a) Sketch of a typical atomic number density. The measured observable is the atom number N given by $n(x)$ integrated over the bin volume (the bins are indicated by the dashed lines). (b) Histogram of probabilities to find N particles in a bin.

III. NONINTERACTING FERMIONS AND BOSONS

We start by recalling some properties of noninteracting Fermi and Bose gases at a given temperature T . Here, individual atoms are independent and we can obtain the statistics as

$$S(\chi) = \mp \sum_k \ln [1 \pm f_{\pm}(k)(e^{i\chi} - 1)], \quad (2)$$

where $f_{\pm}(k) = [\exp((\epsilon_k - \mu)/k_B T) \pm 1]^{-1}$ is the Fermi (Bose) occupation function. The chemical potential is

determined by the average atom number, i. e., $C_1 = \bar{N} = \sum_k f_{\pm}$. Obviously, the number statistics of fermions and bosons differs drastically in the degenerate regime. In the nondegenerate regime f_{\pm} is small and we find the Poissonian statistics of classical particles $S(\chi) = -\bar{N}(\exp(i\chi) - 1)$, both for fermions and bosons.

For degenerate fermions $k_B T \ll \epsilon_F$ the statistics is

$$S(\chi) = -i\chi \bar{N} - (Dk_B T / 4\epsilon_F) \bar{N} \chi^2. \quad (3)$$

Here and in the following, $S(\chi)$ is defined in the interval $[-\pi, \pi]$ and extended periodically. Thus, particle-number fluctuations are suppressed by T/ϵ_F in comparison to the classical case [41]. Remarkably, the statistics is Gaussian and consequently all cumulants C_n for $n \geq 3$ vanish. This behavior resembles fermions in a 1D wire [42] and can be interpreted as a consequence of anti-bunching. Note, that the Gaussian nature of the statistics is a consequence of particle-hole symmetry and is therefore strictly limited to the degenerate regime. Higher-order corrections in $k_B T/\epsilon_F$ will introduce deviations from the Gaussian limit, and lead e.g. to the appearance of higher-order odd cumulants, which are directly related to deviations from the perfect particle-hole symmetry.

In contrast to that, we obtain quite a different behavior for free bosons. Approaching the degeneracy temperature $T_C^{\text{BEC}} = 2\pi\hbar^2 n^{2/3} / m\zeta(3/2)^{2/3}$ [43] from above, the fluctuations are enhanced due to the large factor $f_{-}(k)(f_{-}(k) + 1)$. In the condensed regime, the occupation of the ground state becomes macroscopically large and the grand-canonical approach is no longer valid [44, 45]. This can be seen if we take the limit $T = 0$ of Eq. (2), leading to $S(\chi) = \ln [1 - \bar{N}(e^{i\chi} - 1)]$, which corresponds to a negative binomial distribution and the fluctuations therefore diverge according to $C_n \sim \bar{N}^n$. As in the grand-canonical ensemble the chemical potential is $\mu = 0$ below the critical temperature $T < T_C^{\text{BEC}}$, an arbitrarily large number of bosons can be transferred from the reservoir into the bin, leading to unphysically large fluctuations. In order to find the correct fluctuations in this case, we calculate the FCS from Eq. (1) explicitly. We divide the boson operator into $a_k = b_k + c_k$, where

$$b_k = \int_{V_{\text{bin}}} d^3r e^{ikr} \Psi(r), \quad c_k = \int_{V'} d^3r e^{ikr} \Psi(r), \quad (4)$$

where V' is the volume V without the volume V_{bin} of the bin. We consider a fully condensed state of N_{tot} non-interacting bosons $|\psi\rangle = (a_0^\dagger)^{N_{\text{tot}}} |\text{vac}\rangle = (b_0^\dagger + c_0^\dagger)^{N_{\text{tot}}} |\text{vac}\rangle$. Using Eq. (1) for the bin number operator $\hat{N} = \sum_k b_k^\dagger b_k$ we obtain a binomial statistics for a non-interacting bosonic gas in a bin at $T = 0$

$$S(\chi) = -N_{\text{tot}} \ln [1 + \frac{V_{\text{bin}}}{V} (e^{i\chi} - 1)]. \quad (5)$$

For bin volumes $V_{\text{bin}}/V \ll 1$ the particle-number statistics becomes Poissonian, i.e. $S(\chi) \approx -\bar{N}(e^{i\chi} - 1)$, where $\bar{N} = N_{\text{tot}} V_{\text{bin}}/V$.

IV. BCS GROUND STATE

We now turn to a Fermi gas with an attractive interaction parameterized by a scattering length a . The Hamiltonian for the full system is number conserving and as such shows no number fluctuations at all. Here we consider a bin, i.e. a small subsystem, which we assume to be described by the non-number conserving BCS Hamiltonian [7]. A straightforward ansatz for a non-number conserving many-body state, which takes the pairing interaction into account, is the BCS wave function, which will be used in the following and is known to correctly describe both the BCS and the BEC limit [8, 9, 10]. We will later show that the approach also reproduces the correct counting statistics in the two limits, and therefore prefer to use this transparent, almost analytical approach instead of more complex canonical approaches. The BCS wave function is given by

$$|\text{BCS}\rangle = \prod_k \left(u_k + v_k c_{k\uparrow}^\dagger c_{-k\downarrow}^\dagger \right) |0\rangle. \quad (6)$$

The variational procedure yields $v_k^2 = 1 - u_k^2 = (1 - (\epsilon_k - \mu)/E_k)/2$, where $E_k^2 = (\epsilon_k - \mu)^2 + \Delta^2$ is the energy of quasiparticle excitations. The order parameter Δ and the chemical potential are fixed by the self-consistency equations

$$\Delta = -\lambda \sum_k u_k v_k, \quad \bar{N} = 2 \sum_k v_k^2, \quad (7)$$

where λ is the BCS coupling constant. After renormalization of the coupling constant λ and considering only the low-energy limit, the gap equation can be related to the two-particle scattering amplitude [9, 46].

The product form of the BCS wave function greatly simplifies the calculation of the statistics, since different k states can be treated separately. For a single pair of states ($k \uparrow, -k \downarrow$) the sum over all possible configurations can be easily performed

$$e^{-S_k(\chi)} = \langle \text{BCS} | e^{i(\hat{n}_{k\uparrow} + \hat{n}_{-k\downarrow})\chi} | \text{BCS} \rangle = u_k^2 + v_k^2 e^{2i\chi}. \quad (8)$$

The sum of all states yields the result

$$S(\chi) = - \sum_k \ln [1 + v_k^2 (e^{2i\chi} - 1)]. \quad (9)$$

This is one of the main results of our paper. It is valid in 2 and 3 dimensions; the dimension will only enter into the density of states, when transforming the sum over k into an energy integration via the standard expression

$N_D = m^{D/2} (2\epsilon)^{D/2-1} / 2\pi^{D-1} \hbar^D$ for $D = 2, 3$. It should be noted, that in a strictly two-dimensional system the low-energy scattering amplitude vanishes $\sim -1/\log \epsilon$ and consequently the gap equation (7) shows a logarithmic divergence for $\epsilon \rightarrow 0$. However, for the more realistic situation of a quasi-two-dimensional cloud (i.e., a three-dimensional trapped atomic cloud strongly confined in one dimension), this singularity is eliminated. Results derived for the strictly two-dimensional situation are still valid for the quasi-two-dimensional case, however with the chemical potential μ shifted by the ground-state energy.

We now discuss some limiting cases in which analytical expressions can be obtained. On the BEC side, $\mu < 0$ and $\Delta \ll |\mu|$ leads to $v_k^2 \ll 1$ for all energies and allows to expand the logarithm in Eq. (9). The result is

$$S(\chi) = -\frac{\bar{N}}{2} (e^{2i\chi} - 1), \quad (10)$$

which corresponds to a Poissonian number statistics of *pairs of atoms*. This supports the picture of strongly bound pairs in a coherent state. Note, that the factor of 2 in the exponent leads to exponentially growing cumulants in the fermion number, viz., $C_n = 2^n \bar{N}/2$. We therefore expect strong fluctuations. Remarkably, the result (10) is in accordance with the number statistics of condensed bosons in a bin with volume much less than the total volume, see the expression given after Eq. (5) that was derived using the canonical formalism. Since we are counting single fermions instead of bosons in the present case, the counting field χ is replaced by 2χ , and there is a prefactor 1/2. This agreement is quite remarkable, since the starting point of our approach is the grand-canonical formalism. It is an indication that using the BCS wave function to describe the number fluctuations of a small subsystem works surprisingly well.

On the BCS side the situation is quite different. Here $\mu = \epsilon_F \gg \Delta$ and we obtain

$$S(\chi) = -i\chi\bar{N} - \pi\bar{N}D \frac{\Delta}{4\epsilon_F} (|\cos(\chi)| - 1). \quad (11)$$

We observe that the first term is dominant but contributes only to the first cumulant. The fluctuations come from the second term in Eq. (11) which is smaller by a factor Δ/ϵ_F . Furthermore, similar to the degenerate Fermi gas, the odd cumulants C_n for $n \geq 3$ vanish, which is again a consequence of particle-hole symmetry.

Due to the constant density of states in 2D we can obtain an analytical expression for the CGF for arbitrary μ and Δ , which reads

$$S(\chi) = -\bar{N} \frac{\Delta}{\epsilon_F} \left[\cos(\chi) \text{atan} \left(\frac{2\epsilon_F}{\Delta} e^{i\chi} \right) - \text{atan} \left(\frac{2\epsilon_F}{\Delta} \right) \right] - \frac{\bar{N}}{2} \frac{\mu}{\epsilon_F} \ln [1 + v_0^2 (e^{2i\chi} - 1)]. \quad (12)$$

Here $v_0^2 = (1 + \mu/\sqrt{\mu^2 + \Delta^2})/2$ is the BCS coherence function for $k = 0$. While we do not have an analytical expression in 3D, we expect a similar behavior as in 2D. This will be corroborated later by comparing the numerically obtained cumulants in 3D to the 2D case.

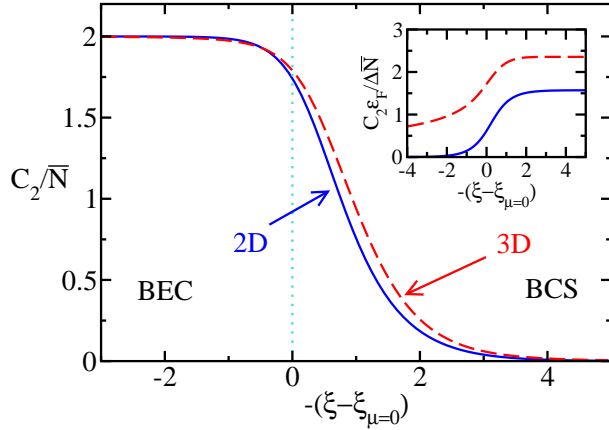


FIG. 2: (Color online) Second cumulant C_2 as a function of $\xi = 1/k_F a$. For the strongly confined, quasi-two-dimensional system (2D), the Fermi vector can be approximated by the inverse of the ground state size: $k_F = \pi/\ell_0$. The dotted line corresponds to $\xi_{\mu=0}$. The inset shows C_2 normalized to Δ/ϵ_F .

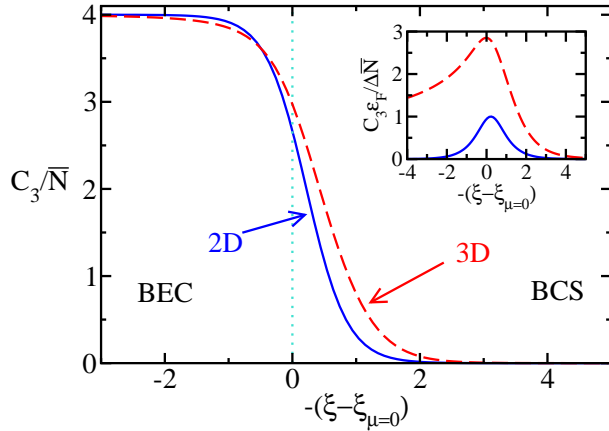


FIG. 3: (Color online) Third cumulant C_3 as a function of ξ as defined in Fig. 2. The dotted line corresponds to $\xi_{\mu=0}$. The inset shows C_3 normalized to Δ/ϵ_F .

In Fig. 2 we show how the density noise changes through the BEC-BCS crossover. Going from the BEC to the BCS regime strongly suppresses the fluctuations in agreement with our previous discussion. The inset shows the fluctuations normalized to Δ/ϵ_F ; they approach a constant value in the BCS limit. Another important information gained from a noise measurement is the order parameter, which can be extracted from a measurement of C_2 in the BCS limit, Eq. (11), as $\Delta/\epsilon_F = 4C_2/\pi\bar{N}D$. Figure 3 shows the third cumulant. The global behavior is rather similar to the second cumulant, i.e. C_3 is strongly reduced going from the

BEC limit to the BCS limit. However, the behavior of C_3 normalized to Δ/ϵ_F shown in the inset is qualitatively different of that shown in the inset of Fig. 2 since C_3 vanishes also in the BCS limit. Note also that C_3 vanishes faster in the 2D case than in the 3D case. To understand the behavior of the third cumulant in more detail we recall that C_3 is related to the skewness of the distribution, i.e. C_3 is a measure of the difference between positive and negative fluctuations. To see this, we note that an elementary binomial event has the property $\ln(1 + v_k^2(\exp(i\chi) - 1)) = i\chi + \ln(1 + u_k^2(\exp(-i\chi) - 1))$. Using this property and the particle-hole symmetry of v_k^2 in the BCS limit, we can rewrite the CGF in the BCS regime as $i\chi N + N_0 \int d\epsilon \ln[1 + v_k^2 u_k^2(\cos(2\chi) - 1)]$. Here we have used that the density of states close to ϵ_F can be approximated by a constant N_0 . The second term is even in χ and therefore only contributes to even cumulants, whereas all odd cumulants for $n \geq 3$ vanish. The difference between 2D and 3D seen in Fig. 3 is caused by the (small) energy dependence of the 3D density of states, which is absent in 2D.

V. FINITE TEMPERATURES

We would now like to discuss the effect of finite temperatures in a qualitative way. The excitations in the BCS theory are fermionic quasiparticles. This is a good approximation on the BCS side of the transition. The fluctuations (measured by the second cumulant C_2) will be reduced with increasing temperature, since they approach the linearly increasing C_2 of the free Fermi gas that is lower at the critical temperature than C_2 at $T = 0$, $C_2^{\text{free}}(T_C^{\text{BCS}}) < C_2(0)$. At even higher temperatures $T \geq T_F$, C_2 reaches the classical Poisson value \bar{N} . The situation on the BEC side of the crossover will be very different. According to our result the statistics is a Poissonian distribution of molecules and, hence, C_2 is doubled in comparison to the classical value for the atomic gas [47]. Upon increasing the temperature, the main effect on the fluctuations will be a breaking up of molecules, which will reduce the second cumulant C_2 above a dissociation temperature to finally reach the value for the classical gas. Thus, we expect quite a different temperature dependence of the number fluctuations on the BEC or the BCS side of the transition.

VI. CONCLUSION

In conclusion, we have calculated the full counting statistics of number densities in an ultracold fermionic atomic cloud with attractive interactions. The number statistics in the vicinity of the BEC-BCS crossover displays interesting features which reveal the nature of the many-body ground state. Poissonian fluctuations of a molecular condensate on the BEC side are strongly suppressed on the BCS side. The size of the fluctuations in

the BCS limit is a direct measure of the pairing potential. We have also discussed the BEC-BCS crossover of the third cumulant and the temperature dependence of the second cumulant. These quantities can be accessed experimentally and provide additional information on the many-body ground state in the crossover regime. The concept of counting statistics in ultracold gases opens interesting possibilities to study the interplay between coherence and correlation in quantum many-body systems.

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